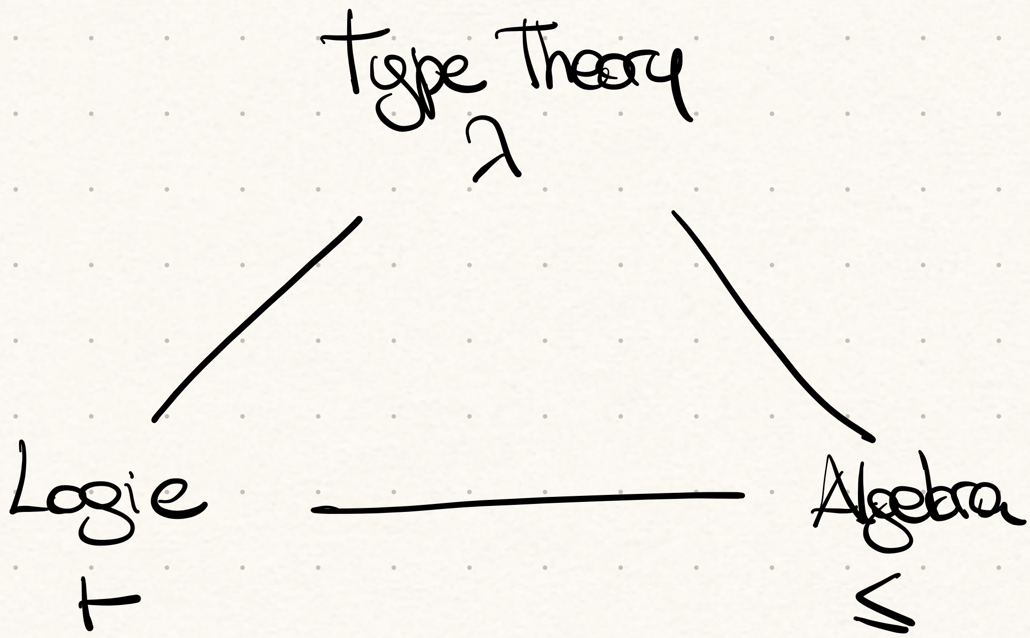
The Computational Trinitarianism



Logic

$$\Gamma \vdash A$$

$$\frac{}{A \vdash A}\,\text{refl} \qquad \frac{A \vdash B \qquad B \vdash C}{A \vdash C}\,\text{cut}$$

Type theory

$$\Gamma \vdash t : A$$

$$\frac{}{x : A \vdash x : A}\,\text{var} \qquad \frac{x : A \vdash t_1 : B \qquad y : B \vdash t_2 : C}{x : A \vdash [t_1/y]\,t_2 : C}$$

(Algebra)

$$\Gamma \leq A$$

Given a preorder $(A, \leq)$

$(A \text{ set} \quad {\leq} \subseteq A \times A)$

$$\frac{}{x \leq x}\,x \in A \qquad \frac{x \leq y \qquad y \leq z}{x \leq z}$$

Let's draw the structure of a preorder

$$\begin{tikzcd} x \arrow[loop right, "\leq"] \end{tikzcd} \qquad \begin{tikzcd} x \arrow[r, bend left, "\leq"] \arrow[rr, bend right, "\leq"'] & y \arrow[r, bend left, "\leq"] & z \end{tikzcd}$$

Category

$$\begin{tikzcd} A \arrow[loop right, "id_A"] \end{tikzcd} \qquad \begin{tikzcd} A \arrow[r, bend left, "f"] \arrow[rr, bend right] & B \arrow[r, bend left, "g"] & C \end{tikzcd}$$

$$(g \circ f) = f ; g$$

$$f \circ id_A = id_B \circ f = f \quad \underline{\text{unitary}}$$

$$h \circ (g \circ f) = (h \circ g) \circ f \quad \text{associativity}$$

$\underline{\text{Set}}$

Objects are sets

Arrows are functions

A, B, C are sets

$Obj(Set)$ "~~set~~ class of sets"

$A \xrightarrow{id} A$

$x \in A$

$id_A(x) = x$

$f : \boxed{A} \to \boxed{B}$ $g : \boxed{B} \to \boxed{C}$

$(g \circ f)$

$$x \longmapsto \underbrace{g(\overbrace{f(x)}^{\in B})}_{\in C}$$

$x \in A$

Poset

Partial Order

$(A, \leq) \qquad \leq \subseteq A \times A$

$\leq$ preorder

$+$

$x, y \in A \qquad x \leq y \wedge y \leq x \Longrightarrow x = y$

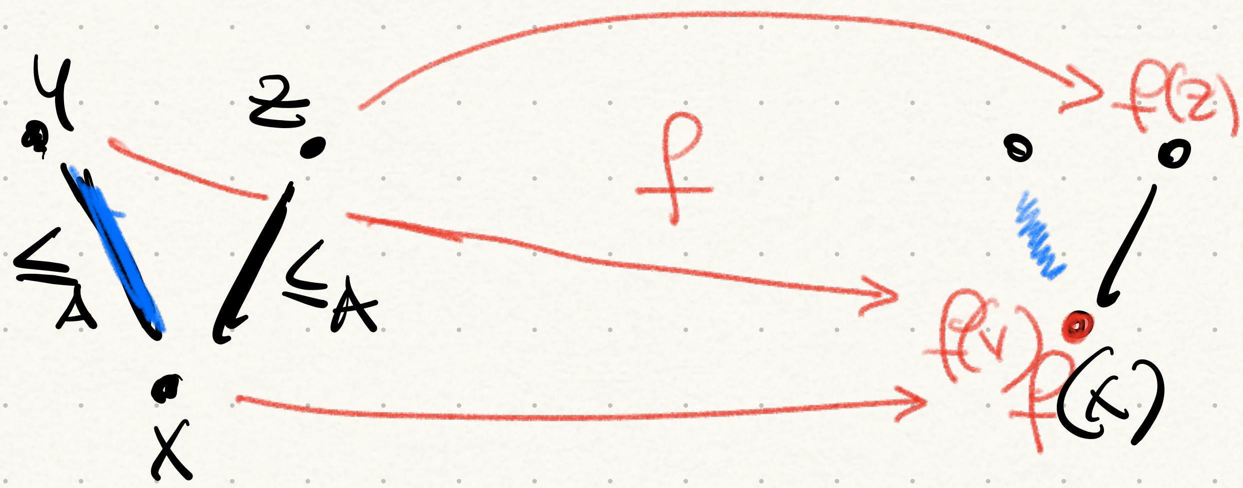
(Antysymmetry)

$0 \leq 1 \leq 2 \leq 3 \; \ldots\ldots$

Poset — Objects are Posets

Arrows $(A, \leq_A)$ $(B, \leq_B)$

$(A, \leq_A)$ $\quad f : A \longrightarrow B \quad$ $(B, \leq_B)$



Preservation of structure

$$x \leq_A y \xrightarrow{f} f(x) \leq_B f(y)$$

monotone function

$(A, \leq_A)$ $(B, \leq_B)$

$\longrightarrow$ $A \cong B$ bijection in Set

$A \ncong B$ in Poset

Isomorphism

$$\begin{tikzcd} A \arrow[loop left, "id_A"] \arrow[r, bend left, "f"] & B \arrow[l, bend left, "g"] \arrow[loop right, "id_B"] \end{tikzcd}$$

$(g \circ f) = id_A$

$(f \circ g) = id_B$

In Set Isomorphism

is a bijection

In Poset Isomorphism

bijection +

that is monotone