$\mathbb{N}, A \times B, A + B, A \to B$

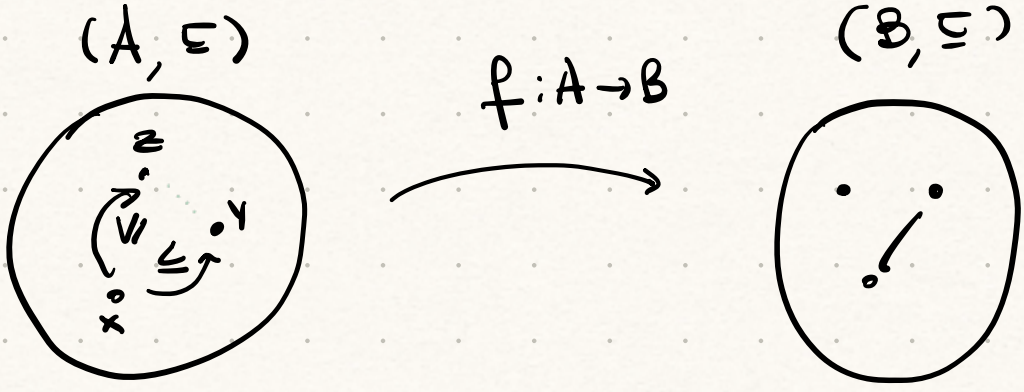
STLC $\longrightarrow$ $\mathbb{C}$

Set of Types τ

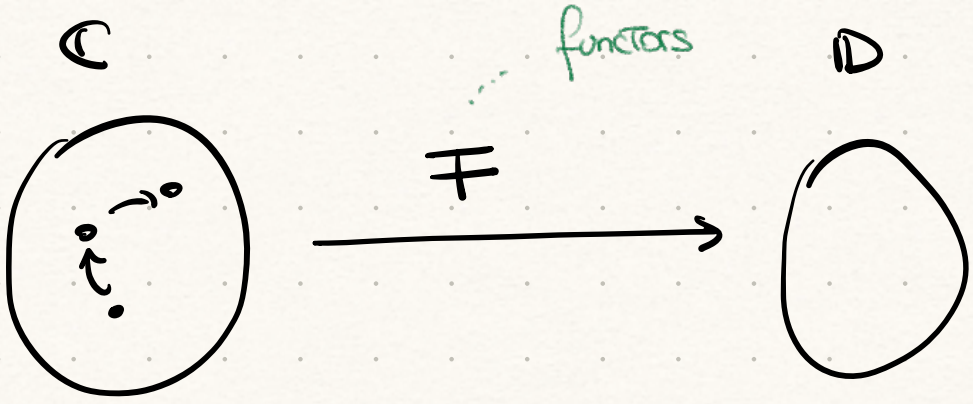
Set of Terms T

$[\![\Gamma \vdash t : \tau]\!] : [\![\Gamma]\!] \longrightarrow [\![\tau]\!]$

Functors



$x \leq y \longrightarrow f(x) \leq f(y)$ monotonicity



$F_0 : Obj(\mathbb{C}) \longrightarrow Obj(\mathbb{D})$ object part

$F_1 : Arr(\mathbb{C}) \longrightarrow Arr(\mathbb{D})$

for $X \in Obj(\mathbb{C})$, FX $\quad f: A \to B \in Arr(\mathbb{C})$ $\quad Ff$ "functorial action"

$$F: \mathbb{C}(A,B) \longrightarrow \mathbb{D}(FA, FB)$$

$FX = \mathbb{C}(S, X)$ **Hauset Functor**

$\mathbb{C}(S,X) \to$ Hauset $\quad f: X \to Y$

$$Ff = \mathbb{C}(S, f): \mathbb{C}(S,X) \longrightarrow \mathbb{C}(S,Y)$$
$$(g: S \to X) \longmapsto f \cdot g$$

A functor is a mapping $F: Obj(\mathbb{C}) \to Obj(\mathbb{D})$ together with a functorial action on arrows s.t

$$F: \mathbb{C}(X,Y) \to \mathbb{D}(FX, FY)$$

and it respects the identity

$$F(id_X) = id_{FX}$$

it respects composition

$$F(f \circ g) = Ff \circ Fg$$

A natural transformation is a family of morphisms

$$\alpha_X : FX \longrightarrow GX$$

F: functor $F : \mathbb{C} \to \mathbb{D}$

G: functor $G : \mathbb{C} \to \mathbb{D}$

$$\alpha : F \longrightarrow G$$

Category $[\mathbb{C}, \mathbb{D}]$ has functors F, G as objects and natural transformations as morphisms