

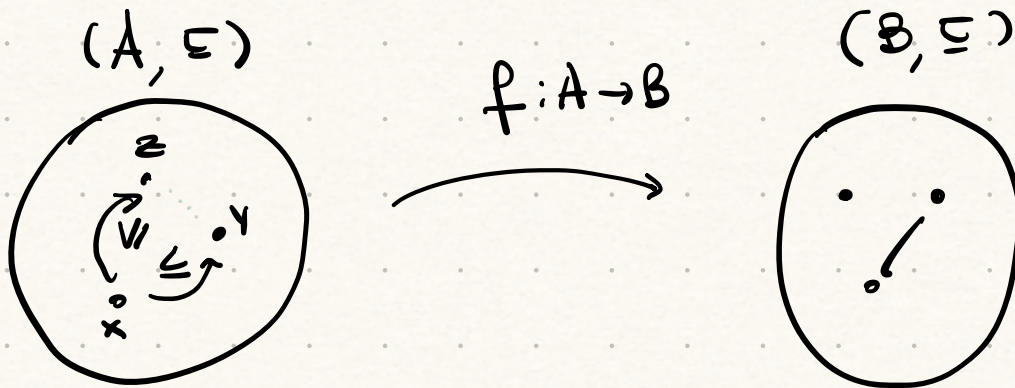
$$\underbrace{\mathbb{N}, A \times B, A + B, A \rightarrow B}_{\text{STLC}} \longrightarrow \mathbb{C}$$

Set of Types τ

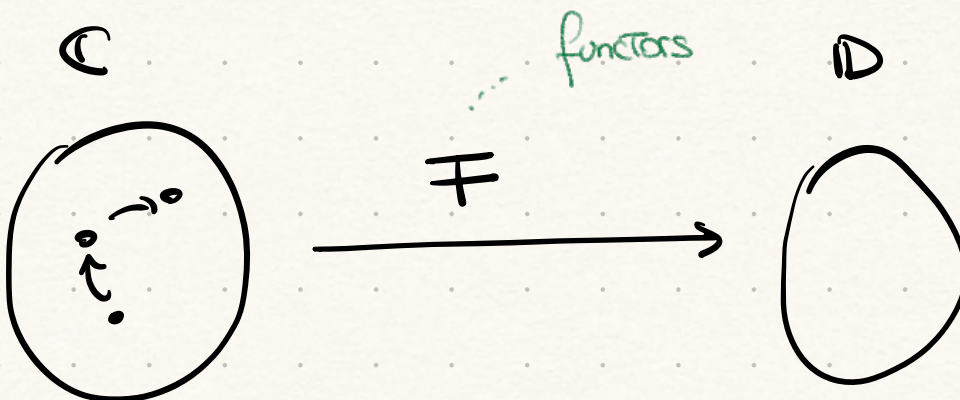
Set of Terms T

$$\llbracket \Gamma \vdash t : \tau \rrbracket : \llbracket \Gamma \rrbracket \rightarrow \llbracket \tau \rrbracket$$

Functors



$$x \leq y \longrightarrow f(x) \leq f(y) \quad \text{monotonicity}$$



$$F_0 : \text{Obj}(\mathbb{C}) \rightarrow \text{Obj}(\mathbb{D}) \quad \text{object part}$$

$$F_1 : \text{Arr}(\mathbb{C}) \rightarrow \text{Arr}(\mathbb{D})$$

for $x \in \text{Obj}(\mathcal{C})$, FX

$f: A \rightarrow B \in \text{Arr}(\mathcal{C})$

Ff

"functorial action"

$$F: \mathcal{C}(A, B) \longrightarrow \mathcal{D}(FA, FB)$$

$$FX = \mathcal{C}(S, x) \quad \underline{\text{Homset Functor}}$$

$\hookrightarrow \text{Homset} \quad f: x \rightarrow y$

$$Ff = \mathcal{C}(S, f): \mathcal{C}(S, x) \longrightarrow \mathcal{C}(S, y)$$

$$(g: S \rightarrow x) \longmapsto f \circ g$$

A functor is a mapping $F: \text{Obj}(\mathcal{C}) \rightarrow \text{Obj}(\mathcal{D})$
together with a functorial action on arrows s.t

$$F: \mathcal{C}(x, y) \longrightarrow \mathcal{D}(Fx, Fy)$$

and it respects the identity

$$F(\text{id}_x) = \text{id}_{Fx}$$

it respects composition

$$F(f \circ g) = Ff \circ Fg$$

A commutative diagram illustrating the functorial action of F on composition. On the left, two arrows f and g are shown, with f above g . Both arrows are curved and point from a common source to a common target. On the right, the corresponding arrows Ff and Fg are shown, also with Ff above Fg and both being curved arrows between the same objects. A horizontal arrow labeled F points from the left side to the right side, indicating the mapping of the entire diagram by the functor F .

A natural transformation is a family of morphisms

$$\alpha_x : FX \longrightarrow GX$$

functor

$$F: \mathcal{C} \rightarrow \mathcal{D}$$

functor

$$G: \mathcal{C} \rightarrow \mathcal{D}$$

$$\alpha: F \longrightarrow G$$

Category $[\mathcal{C}, \mathcal{D}]$ has functors F, G as objects
and natural transformations as morphisms