

A Monad is just (!) a Monoid in the Category of Endofunctors

Definition (Monad)

Endofunctor $T: \mathcal{C} \rightarrow \mathcal{C}$

Unit: $\eta_A: A \rightarrow TA$

Multiplication: $\mu_A: TA \rightarrow A$

- (List; Type $\rightarrow$ Type)

- (return, singleton list)

- "concat"

Construction
of Lists



concat $[e_1, e_2, \dots, e_n] =$
 $e_1 + e_2 + \dots + e_n$
fold return

Definition (Monoid)

M is a set

$1_M \in M$

$(+): M \times M \rightarrow M$

Examples

$(\mathbb{N}, +, 0)$

$(\mathbb{Z}, +, 0)$

~~$(\mathbb{N}, *, 1)$~~

$(\mathbb{Z}, *, 1)$

$(\mathbb{N}, *, 1)$

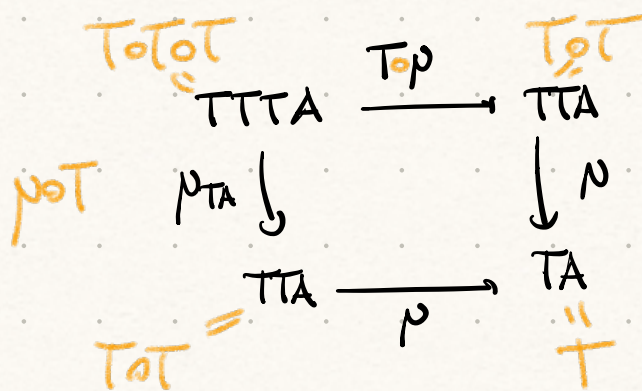
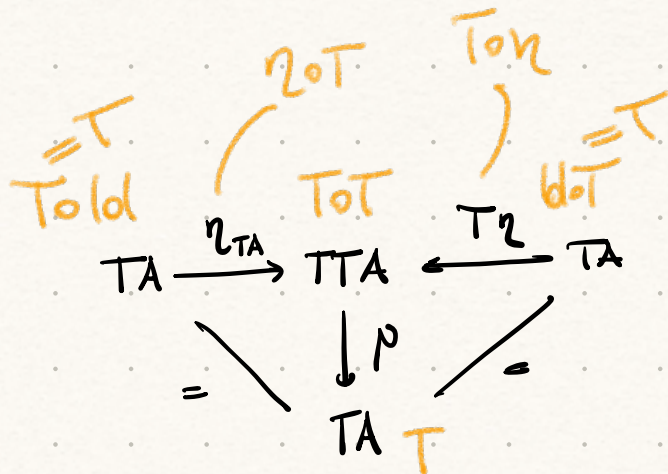
Subject to Equations

$1 + m = m = m + 1$ unitary laws

$(m + n) + r = m + (n + r)$ associativity

Endofunctor category
 $([C, C], \circ, Id)$

$$(\circ): [C, C] \times [C, C] \rightarrow [C, C]$$

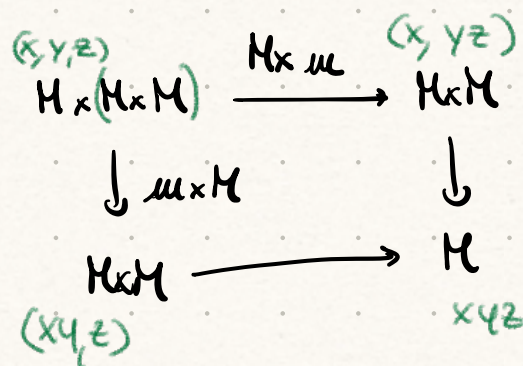
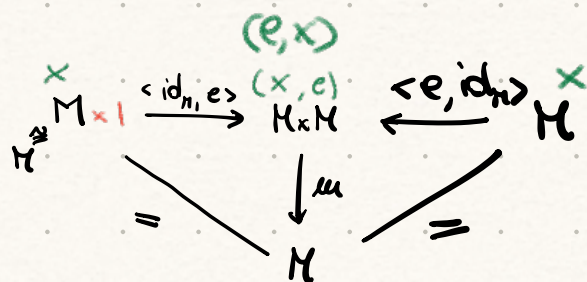


Monoid in a category
 $(C, \otimes, 1)$

it is an object M

$$1 \xrightarrow{e} M \quad \text{unit}$$

$$M \otimes M \xrightarrow{\mu} M \quad \text{multiplication}$$



Happy Summer!

